Seat No. ________________

HAJ-003-2013008

B. Sc. (Sem. III) (CBCS) (W.E.F. 2019) Examination

May - 2023

Maths : Paper - 03 (A)

(Linear Algebra & Real Analysis)

Faculty Code : 003

Subject Code : 2013008

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

Instructions:
(1) All questions are compulsory.
(2) Figures to the right indicate full marks of the question.

1 (a) Answer the following question : **4**

(1) Find constants α and β which satisfy $(3, 5) = \alpha(1, 2) + \beta(2, 3)$.

(2) Define Subspace of a vector space.

(3) Is the set $A = \{(1, 0, 1), (1, 1, 0), (-1, 0, -1)\}$ is linearly dependent? Verify.

(4) Define Linearly Dependent Set of a vector space.

(b) Answer the following question : (any **one**) **2**

(1) Let $W = \{(x, y, 0) / x, y \in R\}$, is subset of R^3 then show that W is subspace of R^3 under vector addition and scalar multiplication.

(2) If $A = \{(1, -2, 5), (2, 1, -1), (3, -1, b)\}$, is Linearly Dependent subset of R^3 then find b.

(c) Answer the following question : (any **one**) **3**

(1) Prove that W_1 and W_2 are sub space of vector space V then $W_1 \cap W_2$ is also a vector space of V.

(2) Check $(1, 2, 4) \in Sp\ A$, where $A = \{(0, 1, -1), (0, 0, 2), (1, 3, 0)\}$.

(d) Answer the following question : (any **one**) **5**

(1) Examine the subset $W = \{(a, b, c) / 2a + 5b - c = 0\}$ is sub spaces of R^3.

(2) Show that the set $V = \{(x, y) / x > 0, y > 0, x, y \in R\}$, is a vector space under usual vector addition and scalar multiplication.

2 (a) Answer the following question : **4**

(1) Define Base of a vector space.

(2) The standard base of $P_3(R)$ is ________.

(3) If $W = Sp\{(1, 1, 0), (0, 1, 1)\}$ then find $\dim W$.

(4) Let W_1 and W_2 be two sub spaces of a finite dimensional vector space V such that $V = W_1 + W_2$ then $\dim V =$ ________.

(b) Answer the following question : (any **one**) **2**

(1) If vector $(m, 3, 1)$ is a linear combination of $a = (3, 2, 1)$ and $b = (2, 1, 0)$ then $m =$ _______.

(2) If $A = \{(2x - 4y + z, x + y, x - y, z) / x, y, z \in R\}$ then find $Sp\ A$.

(c) Answer the following question : (any **one**) **3**

(1) Prove that the set $A = \{(2, 3), (-1, 5)\}$ of R^2 is a Base of R^2.

(2) If $W_1 = \{x - y, y + z, y, z\}$ and $W_2 = \{x, x + y, x + y + z, y - z\}$ are sub spaces of R^4. Find $\dim W_1$, $\dim W_2$.

(d) Answer the following question : (any **one**) **5**

(1) Show that the set $A = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$ is a basis of R^3. Find coordinate of vector $(2, 1, -1)$ with respect to this base.

(2) If W_1 and W_2 be two sub spaces of vector space V then $\dim(W_1 + W_2) = \dim(W_1) + \dim(W_2) - \dim(W_1 \cap W_2)$.

3 (a) Answer the following question : **4**

(1) A vector function $\overline{f}$ is an irrotational then _________.

(2) If $\phi = x^2y + y^2z + z^2x$ then find $\nabla^2\phi =$ ________.

(3) Define divergent of a vector function.

(4) If $\overline{f} = (x^2, y^2, z^2)$ then find $div\,\overline{f}$ at point $(1, 1, 1)$.

(b) Answer the following question : (any **one**) **2**

(1) Find unit normal at the surface $x^2 + y^2 + z^2 = 6$ at point $(2, 1, 1)$.

(2) If $\overline{f} = (x^3, y^3, z^3)$ then prove that

$$grad\,(div\,\overline{f}) = 6\overline{r}, \overline{r} = (x, y, z).$$

(c) Answer the following question : (any **one**) **3**

(1) If $\overline{f} = \left(xy^2z, -x^3y^3, x^2\ yz^3\right)$ then find $curl\ \overline{f}$. Prove that $div\left(curl\ \overline{f}\right) = 0$.

(2) If $\overline{f} = (2x + 3y + az)\,\hat{i} + (bx + 2y + 3z)\,\hat{j} + (2x + cy + 3z)\,\hat{k}$ is irrotational then find a, b, c.

(d) Answer the following question : (any **one**) **5**

(1) Prove in usual notation $grad\ (\phi\psi) = \phi grad + \psi grad\phi$.

(2) Prove in usual notation $div\left(r^n\overline{r}\right) = (n+3)r^n$.

4 (a) Answer the following questions : **4**

(1) Define relation between Cartesian and Cylindrical coordinate.

(2) If $x = r\cos\theta, y = r\sin\theta$, then find $|J|$.

(3) Evaluate : $\int_0^1\int_x^{2x} x^2y\ dy\ dx$.

(4) Evaluate : $\int_0^2\int_0^2 \left(x^2 + y^2\right) dx\ dy$.

(b) Answer the following question : (any **one**) **2**

(1) Evaluate : $\int_0^1\int_0^1 e^{x+y}\ dx\ dy$.

(2) Evaluate : $\iint_R \left(x^2 + 2y\right) dxdy$, where region $R = [0:1, 0:2]$.

(c) Answer the following question : (any **one**) **3**

(1) Evaluate : $\iiint_R xydxdydz$, where curve R is a cube $0 \le x, y, z \le 1$.

(2) Evaluate : $\iint \left(x^2 + y^2\right) dydx$ over the region bounded by $x = 0, x = 1, y = 0, y = x$.

(d) Answer the following question : (any **one**) **5**

(1) Change the order of integration $\int_0^1 \int_{x^2}^{2-x} f(x, y)\, dydx$.

(2) Evaluate : $\iint_R \left(x^2 + y^2\right) dxdy$, where R is bounded by lines $y = x,\ y = -x,\ x - y = 2,\ x + y = 2$.

5 (a) Answer the following question : **4**

(1) Define Gamma function.

(2) State the relation between Beta and Gamma function.

(3) $\Gamma\left(\frac{1}{2}\right)$ = ________.

(4) If the curve C is union of simple curves C_1, C_2 then $\int_C f\, dx$ = ________.

(b) Answer the following question : (any **one**) **2**

(1) Find $\int_{(0,0)}^{(2,2)} y^2\, dx$.

(2) Find $\int (xdy - ydx)$, from (0, 0) to (1, 1) over the straight line $y = x$.

(c) Answer the following question : (any **one**) **3**

(1) Prove $\beta(m, n) = \beta(n, m)$.

(2) If $\overline{F} = \left(x^2 + y\right)\hat{i} + (2x + y)\hat{j}$, then find $\int_C \overline{F} \cdot dr$, where C is line segment joining (2, 0) and (3, 2).

(d) Answer the following question : (any **one**) **5**

(1) State and prove Green's Theorem.

(2) Prove that $\beta(m, n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1 + x)^{m+n}}\, dx$.